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Exlog Weighted Sum Model for long term forecasting

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Abstract

Long-term forecasting of key macroeconomic indicators such as population is very important for future development policy making. Population plays an important role in economic decision making, social security and economic growth. So it is important to develop a good model for predicting economic indicators. In order to improve the growth model, we introduce a new model called Exlog Weighted Sum Model for predicting macroeconomic indicators. This model combines both exponential and logistic models. The proposed model was tested for predicting Mongolian population up to 2040.

Introduction

Population growth is described as a function of time usually by dynamic models based on differential equations. The most common practical methods are component, exponential and logistic models[1].

On the other hand, the population growth can be considered as stochastic variables since population depends on social and economic policies, political stability and so on.

In this paper, first we examine the existing growth models such as exponential, logistic and stochastic models. Then we deal with two classic models. It is well known that exponential and logistic functions have some drawbacks in terms of too fast growth or slow growth in a long term prediction in economy. For this purpose, we introduce a new model called Exlog Weighted Sum Model for predicting macroeconomic indicators and focus on parameter estimation problem of the exponential and logistic models. Finding a parameter of the exponential model reduces to unconstrained minimization problem which has been solved analytically. As far as the parameter of logistic model is concerned, the least square method reduces to difficult nonconvex optimization problem but we have proposed simple formula for finding parameters of a new model.

Methods and Materials

Exlog Weighted Sum Model

Using the exponential and logistic models, we introduce so-called Exlog Weighted Sum Model given by the following formula:

$$y(t) = \lambda_1 y_0 e^{rt} + \lambda_2 \frac{M y_0}{y_0 + (M - y_0)e^{-Mkt}} \qquad (7)$$

where,

$y(t)$: Macroeconomic indicator,

$y(0)$: initial value of y at the moment $t = 0$,

λ_1, λ_2 are parameters of the model, and $\lambda_1 + \lambda_2 = 1$, $\lambda_1 > 0, \lambda_2 \geq 0$.

t	t_1	t_2	**…**	t_m
y	y_1	y_2	**…**	y_m

Using the above data, we can estimate the parameters of the model by solving the following constrained minimization problem:

$$F(\lambda_1, \lambda_2) = \sum_{t=1}^{m} \left[\lambda_1 y_0 e^{rt} + \lambda_2 \frac{M y_0}{y_0 + (M - y_0)e^{-Mkt}} - y_i\right]^2 \to min \qquad (8)$$

$$\lambda_1 + \lambda_2 = 1, \lambda_1 > 0, \lambda_2 \geq 0 \qquad (9)$$

In general, problem (8)-(9) is convex optimization problem[5]. For simplicity, we assume that we have the following models

$$y^{ex} = y_0 e^{rt}, \quad y^{log} = \frac{M y_0}{y_0 + (M - y_0)e^{-Mkt}}$$

and data:

t	y_i^{ex}	y_i^{log}	y_i
t_0	y_0^{ex}	y_0^{log}	y_0
t_1	y_1^{ex}	y_1^{log}	y_1
t_2	y_2^{ex}	y_2^{log}	y_2
…	…	…	…
t_m	y_m^{ex}	y_m^{log}	y_m

Formula (7) has the form

$$y = y^{ex} + y^{log}.$$

Here, values of y_i^{ex} and $y_i^{\log}$ have been estimated by exponential and logistic functions.

Then finding parameters of the *Exlog Weighted Sum Model* reduces to a constrained minimization problem:

$$F(\lambda_1, \lambda_2) = \sum_{t=1}^{m} \left[\lambda_1 y_i^{ex} + \lambda_2 y_i^{log} - y_i\right]^2 \to min \qquad (10)$$

$$\lambda_1 + \lambda_2 = 1, \lambda_1 \geq 0, \lambda_2 \geq 0 \qquad (11)$$

Since the problem is convex, then we apply Lagrange method. The Lagrangean function is

$$L(\lambda_1, \lambda_2, \lambda) = \sum_{t=1}^{m} \left[\lambda_1 y_i^{ex} + \lambda_2 y_i^{log} - y_i\right]^2 + \lambda(\lambda_1 + \lambda_2 - 1)$$

Where λ is a Lagrange multiplier.

$$\begin{cases} \lambda_1 = \dfrac{\sum_{i=1}^{m} [y_1 y_i^{ex} - y_i^{ex} y_i^{log} - y_i y_i^{log} + (y_i^{log})^2]}{\sum_{i=1}^{m} (y_i^{ex} - y_i^{log})^2} \\ \lambda_2 = 1 - \lambda_1 \end{cases} \qquad (12)$$

The values of λ_1, and λ_2 were computed by (12) and found as $\lambda_1 = 0.8$, and $\lambda_2 = 0.2$. Formula (12) can be used in predicting some other macroeconomic indicators.

Results

Forecasting Mongolian Population Growth

Traditional exponential model provides too fast growth, on the other hand, the logistic model ensures too slow growth which are not suitable for long term growth of considered indicators. We illustrate Exlog Weighted Sum Model on Mongolian Population data. Data covers period between 2000 and 2020. In model (7) we take values of $M = 5{,}000.0$ and k, r computed by formulas (3), (6) as follows:

$$k = 0.0000079, \; r = 0.0167.$$

Based on survey for the period 2000-2020[6], we predict the Mongolian population by Exlog Weighted Sum Model:

$$y = 1926e^{0.0167t} + \frac{2407500}{2407.5 + 2592.5e^{-0.0395t}}$$

We forecast the Mongolian population up to 2040 year in the Table 2.

Table 1. Mongolian population data from 2000 to 2020.

Year	Thousand people
2000	2,407.5
2001	2,442.5
2002	2,475.4
2003	2,504.0
2004	2,533.1
2005	2,562.4
2006	2,583.2
2007	2,620.9
2008	2,665.9
2009	2,716.3
2010	2,761.0
2011	2,811.7
2012	2,867.7
2013	2,930.3
2014	2,995.9
2015	3,057.8
2016	3,119.9
2017	3,177.9
2018	3,238.5
2019	3,296.9
2020	3,357.5

Table 2. Mongolian population up to 2040 year (thousands of people)

Year	y^{ex}	y^{log}	y^{exlog}
2021	3424.3	3403.1	3420.1
2022	3482.2	3445.8	3474.9
2023	3541.1	3487.8	3530.5
2024	3601.1	3529.2	3586.7
2025	3662.0	3569.9	3643.6
2026	3723.9	3610.0	3701.1
2027	3786.9	3649.3	3759.4
2028	3851.0	3688.0	3818.4
2029	3916.2	3725.9	3878.1
2030	3982.4	3763.1	3938.5
2031	4049.8	3799.5	3999.7
2032	4118.3	3835.2	4061.7
2033	4188.0	3870.2	4124.4
2034	4258.8	3904.4	4187.9
2035	4330.9	3937.9	4252.3
2036	4404.1	3970.6	4317.4
2037	4478.7	4002.6	4383.4
2038	4554.4	4033.8	4450.3
2039	4631.5	4064.2	4518.0
2040	4709.8	4093.9	4586.7

Conclusions

We examine the existing population growth models such as exponential and logistic models and propose a method for finding parameters of the models by solving corresponding convex and nonconvex optimization problems.

We propose also so-called Exlog Weighted Sum Model combining well known classical models such as exponential and logistic for predicting macroeconomic indicators of Mongolia. We derive formulas for estimating weights of the proposed model by solving a constrained convex minimization problem. The proposed model was tested for predicting Mongolian population up to 2040. The proposed approach can be applied to forecasting any macroeconomic indicators.

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