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Optimal Advertising Expenditure

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YOUR LOGO

Abstract

We used Optimal Control Theory to better understand when a company should maximize their advertising and marketing, as well as when they should end their advertising to maximize their profit. This problem was approached utilizing the method of dynamic optimization. Using the Maximum Principle, we derive the necessary conditions for optimality. Then, through research we plan to use accurate real-world data in order to derive the values for our parameters. This will allow us to find an optimal advertisement plan that can be used to help real companies advertise optimally.

Introduction

- How much should a company invest in advertising?
  - Multiple factors which affect the answer
  - Cost to advertise?
  - Customer response & audience to reach
  - Not always a clear line on how much to spend
- Use Optimal Control Theory to help better understand when a company should maximize their marketing & maximize their profit.
  - Dynamic optimization
  - Maximum Principle to derive the necessary conditions for optimality.
- Factors entering into marketing & advertising decisions:
  - The advertising method & advertisement time frame for the product.
  - Budget for advertising campaign
- Need for a plan in place to optimize the advertising cost & opportunities of the product while maximizing net profitability

The Model

We plan to maximize our profit from advertising from time 0 to T, hence we need to integrate our function from 0 to T, with respect to t.

$$\text{Max}_a J = \int_0^T \left(\pi\left(\frac{r}{M}(1-x)a^k\right) - ca\right) e^{-\rho t} dt$$

subject to

$$x' = \frac{r}{M}(1-x)u - \beta x$$

$a(t)$ – Advertisement effectiveness
$s(t)$ – Number of people who own our product
M – Market Potential (Total number of people able to buy product)
$x(t) = \frac{s(t)}{M}$ – Market share, portion of market which bought/owns product
$x'(t)$ – Rate of change of our market share, new people who own our product
r – Reaction from customers for each advert level
c – Price per unit of advertisement
ρ – Discount rate
k – A reaction function that measures the power of our advertisement, from (0,1)
$u(t) = a^k$ – Function related to advert effectiveness (used in order to simplify our problem)
β – Decay factor
$T = \frac{1}{\beta}$ – Lifetime of our item
$\lambda(t)$ – Current shadow value
π – Expected profit
$ca(t)$ – Total cost of advert, where c is cost per unit effort

Model - continued

In order to simplify our function, we let $u = a^k \implies a = u^{\left(\frac{1}{k}\right)}$

Substituting this in the in the integral, we can reformulate our maximization problem as

$$\text{Max}_a J = \int_0^T \left(\pi\left(\frac{r}{M}(1-x)u\right) - cu^{\left(\frac{1}{k}\right)}\right) e^{-\rho t} dt$$

And notice that $x' = \frac{r}{M}(1-x)u - \beta x \implies x' + \beta x = \frac{r}{M}(1-x)u$

Hence, we can replace $\frac{r}{M}(1-x)u$ in our equation with $x' + \beta x$ to get:

$$\text{Max}_a J = \int_0^T (\pi(x' + \beta x) - cu^{\left(\frac{1}{k}\right)}) e^{-\rho t} dt$$

The current value Hamiltonian corresponding to our problem is

$$H(x,u,\lambda) = F(x,u) + \lambda\left(\frac{r}{M}(1-x)u - \beta x\right)$$

where λ is the Lagrangian multiplier or shadow value, and

$$F(x,u) = \pi(\rho+\beta)x - cu^{\left(\frac{1}{k}\right)}$$

Model Solution

The following three equations can be potentially solved for the optimal trajectory of advertisement effort, market share, and marginal advertisement.

$$-\frac{cu^{\left(\frac{1}{k}\right)}}{ku} + \frac{\lambda r(1-x)}{M} = 0 \implies \lambda = -\frac{cu^{\frac{-k+1}{k}} M}{kr(-1+x)}$$

$$\implies \frac{d}{dt}\left(-\frac{cu^{\frac{-k+1}{k}} M}{kr(-1+x)}\right) - \left((\rho)\left(-\frac{cu^{\frac{-k+1}{k}} M}{kr(-1+x)}\right)\right) = -\left(\pi(\beta+\rho) + \lambda(-\frac{ru}{M} - \beta)\right)$$

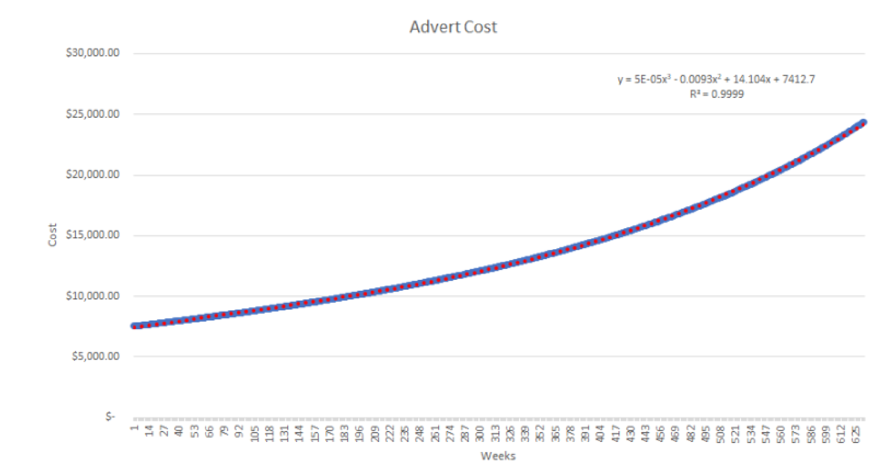
$$x' = \frac{r}{M}(1-x)u - \beta x$$

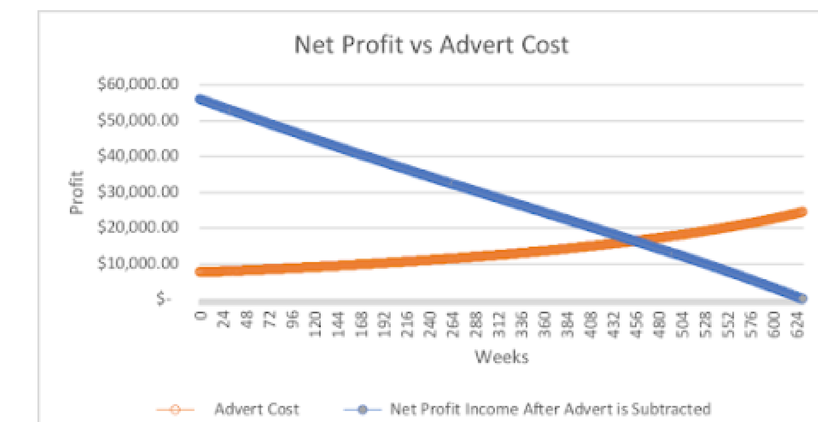
Empirical illustration

We collected data from a research study that looked at the impact of advertising across Breweries in Nigeria. The study recorded 6 years worth of financial summaries between 2006 to 2011. It looked at the sales turnover, profit before tax, tax, profit after tax, and the amount of advertising of these breweries.

$k = \frac{1}{2}$(A reaction function that measures the power of our advertisement)
$\rho = 0.023$(Current discount rate we get directly from the federal reserve)
$\beta = \frac{1}{100}$(Lifetime of our item)
$c = 1.9268$(Price per unit of advertisement)
$\pi = 70000$(Expected profit)
$r = 0.0899$(Reaction from customers for each advert level)
$M = 1$(Normalized to 1 in order to simplify our results)

Empirical Model





Using the data we collected from the research study we were able to compute numerical values for the net profit of the company before subtracting advertisement costs, the net profit after subtracting advertisement costs, and advertisement costs. With the use of numerical methods, we were able to observe the changes over a period of 631 weeks, which is just over the span of 12 years. Our calculations show that the net profit before advertisement costs are subtracted is equal to a total of $26, 289, 585.60. This net profit decreases from $63, 484.61 to $24, 441.64 due to factors including consumers buying the product less each year. As the net profit before the advertisement is subtracted is decreasing, we noticed that the cost of the advertisement has an opposite effect. This cost gradually increases each week. The initial cost of advertising is $7, 528.86 whereas in week 631 it costs $24, 392.71. This results in a total of $8, 565, 803.25 over the course of 12 years. The net profit after all advertisement costs are subtracted shows the same downward curve as before the advertisement costs are subtracted. The net profit starts at $55, 955.74 and continues to decrease to $48.93 by the 631st week. The net profit results in a total of $17, 723, 782.36 after the 12 years. In week 456 we computed the cost of advertising to be $16, 297.76 however the net profit af- ter the advertisement costs are subtracted resulted in only $16, 236.19. This shows a loss of $61.57. The difference continues to be negative values for the remainder of our calculations. By the 12th year there is a deficit of $24, 343, 78.

Conclusions

Using the Maximum Principle, we were able to derive the necessary conditions that must be satisfied in order to search for optimal solutions. Through our research we managed to find information regarding most of our parameters from a paper which discussed the impact of advertising the sales turnover of Breweries in Nigeria. Analyzing this data, we settled on values for our parameters. Finally, using maple we were able to solve this system of highly nonlinear necessary conditions and use numerical methods in order to generate results.

Contact

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