

### Abstract

Electrical Impedance Tomography (EIT) is an imaging technique to visualize admittivity or impedance distribution inside an electrically conducting object both in 2D and in 3D.

In frequency difference EIT problem measured data at two different frequency subtract to produce images of changes in admittivity distribution with respect to frequency, which are acquired at the same time.

We consider admittivity distribution

$$\gamma = \sigma + i\omega\epsilon$$

as a function position  $r(x,y,z)$ .

Where,

$\gamma$  - admittivity

$\sigma$  - electrical conductivity

$\omega$  - angular frequency

$\epsilon$  - electrical permittivity

### Introduction

We attached E electrodes on the surface of interested area  $\Omega$ .

Then inject current between a selected pair of electrodes  $\mathcal{E}_j$  and  $\mathcal{E}_{j+1}$  for  $j = 1, 2, \dots, E-1$  and E with  $\mathcal{E}_{E+1} = \mathcal{E}_1$ .

The induced electrical potential denote by  $u^j$  in the domain  $\Omega$  and potential  $u^j$  satisfies the following elliptic PDE with Neumann boundary condition

$$\begin{cases} -\nabla \cdot \gamma \nabla u^j(r) = 0 & \text{in } \Omega \\ \int_{\mathcal{E}_j} \gamma \frac{\partial u^j}{\partial n} dr = - \int_{\mathcal{E}_{j+1}} \gamma \frac{\partial u^j}{\partial n} dr \\ -\gamma \nabla u^j n = 0 & \text{on } \partial\Omega \setminus \mathcal{E}_j \cup \mathcal{E}_{j+1} \end{cases} \quad (1)$$

where, n is outward normal vector on  $\partial\Omega$ .

Inject current through E pair of electrodes and measure induced voltage on the E voltage electrodes and we get  $E^2$  measurements of voltages which are expressed in a matrix form F for a given  $\gamma$ .

$$F(\gamma) = [V^{1,1}(\gamma), \dots, V^{1,E}(\gamma), V^{2,1}(\gamma) \dots V^{2,E}(\gamma), \dots, V^{E,1}(\gamma), \dots, V^{E,E}(\gamma)]$$

### Methods and Materials

We inject same currents into the imaging domain with  $\gamma$  and a known  $\gamma^0$ .

**Lemma .** The perturbation  $\delta\gamma = \gamma - \gamma^0$  satisfies

$$\int_{\Omega} \delta\gamma \nabla u^j \cdot \nabla u_0^k dr = f^l(\gamma) - f^l(\gamma^0) \quad (2)$$

where,  $u_0^k$  is the solution of (1) with  $\gamma^0$  in place of  $\gamma$  and  $f^l(\gamma)$  is lth component of  $F(\gamma)$ .

We discretize the domain  $\Omega$  into N subregions as  $\Omega = \bigcup_{n=1}^N T_n$ . We assume  $\gamma, \gamma^0$  and  $\delta\gamma$  are constants in each  $T_n$ . Then we can express (2) as  $S_{\gamma^0} \delta\gamma = \delta F$

or

$$\begin{pmatrix} \int_{T_1} \nabla u_0^1 \nabla u_0^1 & \int_{T_2} \nabla u_0^1 \nabla u_0^1 & \dots & \int_{T_N} \nabla u_0^1 \nabla u_0^1 \\ \int_{T_1} \nabla u_0^1 \nabla u_0^2 & \int_{T_2} \nabla u_0^1 \nabla u_0^2 & \dots & \int_{T_N} \nabla u_0^1 \nabla u_0^2 \\ \vdots & \vdots & \ddots & \vdots \\ \int_{T_1} \nabla u_0^E \nabla u_0^E & \int_{T_2} \nabla u_0^E \nabla u_0^E & \dots & \int_{T_N} \nabla u_0^E \nabla u_0^E \end{pmatrix} \begin{pmatrix} \delta\gamma_1 \\ \delta\gamma_2 \\ \vdots \\ \delta\gamma_N \end{pmatrix} = \begin{pmatrix} f^1(\gamma^0 + \delta\gamma) - f^1(\gamma^0) \\ \vdots \\ f^E(\gamma^0 + \delta\gamma) - f^E(\gamma^0) \\ \vdots \end{pmatrix}$$

Solving the linear equation

$$\delta\gamma^0 = (S_{\gamma^0}^T S_{\gamma^0} + \lambda R)^{-1} S_{\gamma^0}^T \delta F^0$$

where,  $\lambda$  is the a regularization parameter and R is a regularization matrix.

Algorithm

Step 1. Let  $\gamma^0$  as the initial guess and  $\delta$  is tolerance.

Step 2. For  $\gamma^m$  solve the forward problem (1) and get  $u_{\gamma^m}^j$

Step 3. Compute  $S_{\gamma^m}$  matrix

Step 4. Calculate  $\delta\gamma^m$

Step 5. Update  $\gamma^{m+1} = \gamma^m + \delta\gamma^m$

Step 6. Repeat the process until  $\|\delta\gamma^m\| < \delta$ .

### Results

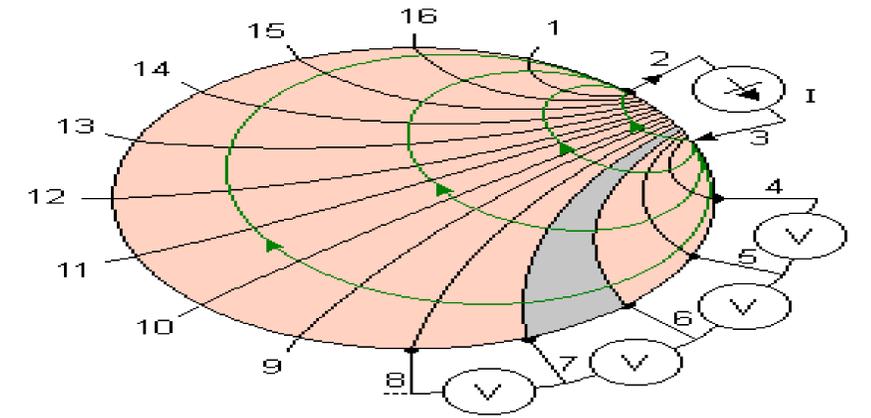


Figure 1. NtD data from 16- channel EIT system.

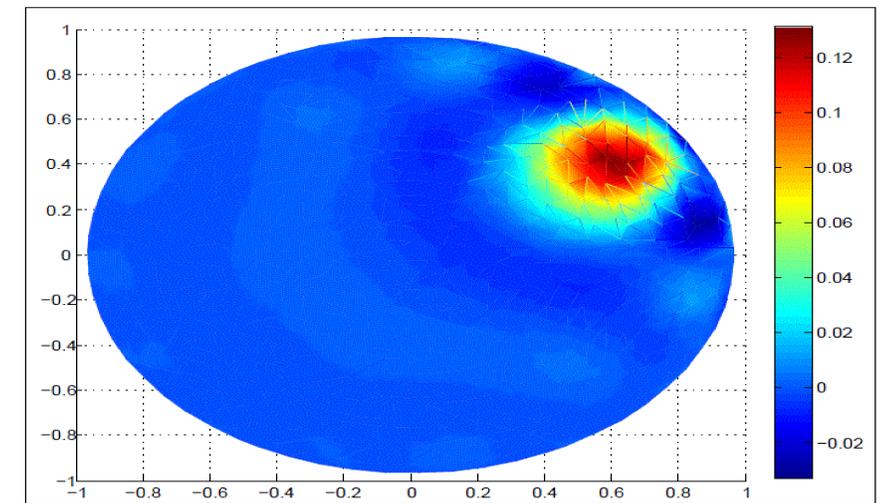


Figure 2. Reconstructed image

### Conclusions

Inverse problem of EIT inherits some technical difficulties, suffering from the ill-posed problem with geometry, limited electrode measurements, and were affected by unknown boundary uncertainty in electrode position, and systematic artifacts, even though it is able to show frequency-difference imaging by non-destructive way.

### Contact

[Uranchimeg Sangas]  
[Mongolian University of Science and Technology]  
[MUST, Baga toiruu, 8th khoroo, Sukhbaatar district, Ulaanbaatar, Mongolia]  
[uranchimeg.s@must.edu.mn]

### References

1. Lawrence C.Evans Partial Differential Equations Volume 19 American Mathematical Society Providence, Rhode Island
2. Walter A. Strauss Partial Differential Equations Brown University, John Wiley & Sons, Ltd
3. Oden Demkowicz Applied Functional Analysis, ICES The University of Texas at Austin
4. Moon Kyung Choi Characterization of shape and position errors in the linearized EIT reconstruction methods, Yonsei University