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Optimal Portfolio Selection of Currencies

Gantigmaa.G[1]; Davaajargal.J[2]

Institute of Mathematics and Digital Technology, Mongolian Academy of Sceinces

Abstract

We consider optimal portfolio selection problem based on Markowitz model [1]. The problem reduces to the following two variants:
1. Risk minimization
2. Return maximization
First variant is formulated as a convex minimization problem while second one belongs to a class of fractional programming. The return maximization problem is based on Sharpe Ratio for defining the return per unit of risk. For solving the risk minimization problem [2] the conditional gradient method is used. In order to solve the second variant, we apply the Dinkelbach Algorithm [3] developed for solving fractional programming. Numerical experiments have been done on a data of 6 different currency returns which are used commonly in Mongolian foreign trade.

Introduction

Currency is any form of money in circulation to the public. Exchange rate, on the other hand, refers to the price of one currency expressed in terms of another, and in this work, the currency is considered to be the currency of a nation or an economic region.
Mongolia's economy opened up in 1990, and the stability of the commercial banking system has played an important role in the sustainable development of the national economy. Funds raised by commercial banks are made up of multinational currencies, allowing banks to make a profit or reduce risk to a reasonable level by managing the risk of exchange rate fluctuations in those currencies.
This paper aims to demonstrate the application of Markowitz's theory and Dinkelbach Algorithm in the formation of a commercial bank's currency portfolio, and aims to identify portfolios that minimize exchange rate risk and maximize returns.
Due to the fact that commercial banks provide payment services, they are required to serve all widely used currencies in Mongolia. Therefore, the "long position" was included in the portfolio selection with minimum risk to include all widely used currencies, which differed from previous studies.

Data

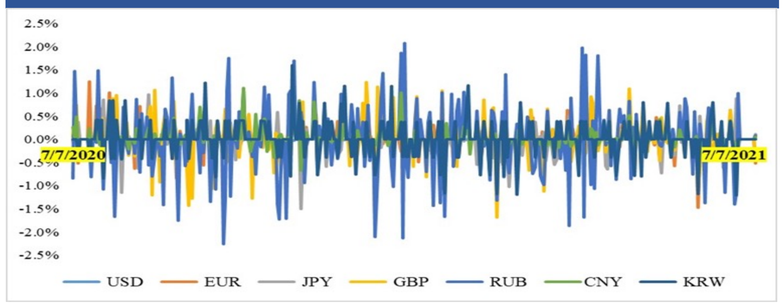

Figure 1. Fluctuations in the daily return of currencies commonly used in Mongolia's foreign trade [4].

Methods

1. Risk minimization

The following optimization problem will be considered to find the minimum risk portfolio of currency investment.

$$\begin{cases} min\ \sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j \\ \sum_{i=1}^n x_i = 1 \end{cases} \quad (1)$$

Here: σ_p^2- the variation of the total return of the portfolio; σ_{ij}- the covariance of return on currencies i and j; x_i- the percentage of return on currencies to the total return.

The Lagrange equation for finding the solution is as follows [2].

$$L(x,\lambda) = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j - \lambda(\sum_{i=1}^n x_i - 1) \quad (2)$$

To find the optimal solution from above function, we equate the partial derivatives of $x_i$and λ to 0. Here λ is the Lagrange multiplier. This problem can be solved with Microsoft Excel Solver.

2. Return maximization

The following optimization problem will be considered to find the maximum return portfolio of currency investment.

$$\begin{cases} max\ f(x) = \frac{f_1(x)}{f_2(x)} = \frac{\sum_{i=1}^n \bar{r_i} x_i - R_f}{\sigma_p} = \frac{(\sum_{i=1}^n \bar{r_i} x_i - R_f)^2}{\sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j} \\ \sum_{i=1}^n x_i = 1 \\ x_i > 0 \end{cases} \quad (3)$$

Here: $f(x)$- the Ratio of Sharpe; σ_p- standard deviation of the portfolio; $\bar{r_i}$ - average return on investment; $\mathrm{R_f}$ - risk free return and it is a fixed number.

Dinkelbach's algorithm [3] :

Step 0: Choose a feasible point $x_0 \in x$, such that $q_0 = \frac{f_1(x_0)}{f_2(x_0)}$ and let k=0;

Step 1: Solve the problem $f(q_k)$=max$\{f_1(x) - q_k f_2(x)\}$. Let x_{k+1} be a solution to this problem, i.e.

$$\begin{cases} max\ f(x) \\ \sum_{i=1}^n x_i = 1 \\ x_i > 0 \end{cases} \quad (4)$$

Step 2: If $f(q_k)$=0, then x_k is an optimal solution to the problem (4) and stop.

Otherwise, set q_{k+1}=$\frac{f_1(x_k)}{f_2(x_k)}$, $k = k + 1$and go to step 1.

Results

Currency	At the min σ^2	At the max $f(x)$
USD	0.996	0.999
CNY	0.005	0.138e-10
EUR	-0.017	0.361e-10
GBP	-0.002	0.413e-10
JPY	0.013	0.271e-10
RUB	0.003	0.681e-10
KRW	0.001	0.436e-10
Total	**1**	**1**
Final objective	**0.387e-9**	**0.488**

Table 1. Share of currencies in the minimum risk and maximum return portfolios.

Discussion

Placing 99.61% of the investment in the US dollars, creating a long positions in euros and pounds, and short position in other currencies would bring a minimum risk of 0.02%(standard deviation). This is because the US dollar, yen and yuan have the lowest standard deviations, while the ruble has negative covariance against the US dollar, which accounts for the largest percentage of the portfolio.
In order to form the portfolio with the maximum return per unit of risk, also the majority of (99.99%) investments will be invested in US dollars, with a Sharpe Ratio of approximately 0.5. As for US dollar, the main contributing factor was its lowest average return and risk. The yuan and pound have higher average return but also have higher risk compared to US dollar.

Conclusions

In 2020-2021, most countries have implemented a wide range of fiscal policy measures to mitigate the effects of the pandemic and maintain economic stability. Therefore the USD/MNT exchange rate was the most stable.
Zoom out to 100% to preview what this will look like on your printed poster.
This has led to US dollar dominant minimum risk and maximum return portfolios. Rising inflation, and its exceeding policy rate indicate that all investments in the current economy are at some risk.

Contact

[Gantigmaa.G]
[*Institute of Mathematics and Digital Technology, Mongolian Academy of Sceinces*]
[2nd building of the Academy of Sciences, 54B Peace Avenue, Bayanzurkh District, Ulaanbaatar 13330, Mongolia]
[gantigmaa_g@mas.ac.mn]
[(976) 80335656]

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